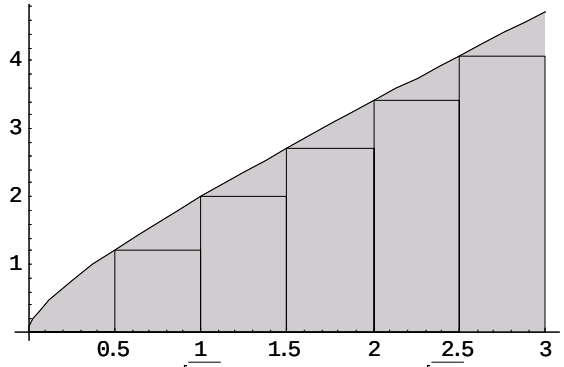
Scheda di laboratorio svolta con le approssimazioni dell'area di un trapezoide con i metodi dei rettangoli e dei trapezi

Ricordando che

$$R_n = \frac{b-a}{n}\sum_{i=0}^{n-1} f\left(a + i\frac{b-a}{n}\right) \text{ e che } T_n = \frac{b-a}{2n}\left(f(a) + f(b) + 2\sum_{i=1}^{n-1} f\left(a + i\frac{b-a}{n}\right)\right)$$

si propongono i seguenti esercizi risolti:

$x + \sqrt{x}$

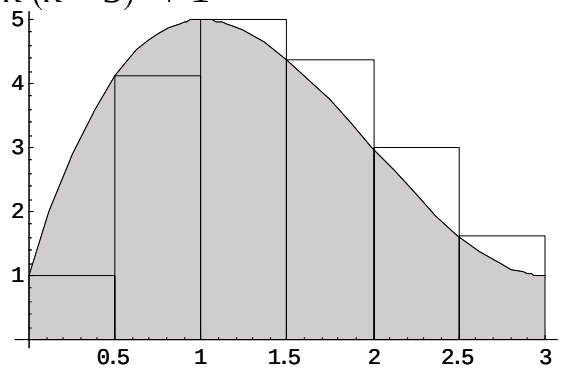


$$R6 = \frac{1}{2}\left(\frac{17}{2} + \sqrt{\frac{3}{2}} + \frac{1}{\sqrt{2}} + \sqrt{2} + \sqrt{\frac{5}{2}}\right) \approx 6.7136$$

Tabella dei valori

x	f(x)	N(f(x))
0	0	0
$\frac{1}{2}$	$\frac{1}{2} + \frac{1}{\sqrt{2}}$	$\frac{1}{2} + \frac{1}{\sqrt{2}}$
1	2	2
$\frac{3}{2}$	$\frac{3}{2} + \sqrt{\frac{3}{2}}$	$\frac{3}{2} + \sqrt{\frac{3}{2}}$
2	$2 + \sqrt{2}$	$2 + \sqrt{2}$
$\frac{5}{2}$	$\frac{5}{2} + \sqrt{\frac{5}{2}}$	$\frac{5}{2} + \sqrt{\frac{5}{2}}$
3	$3 + \sqrt{3}$	$3 + \sqrt{3}$

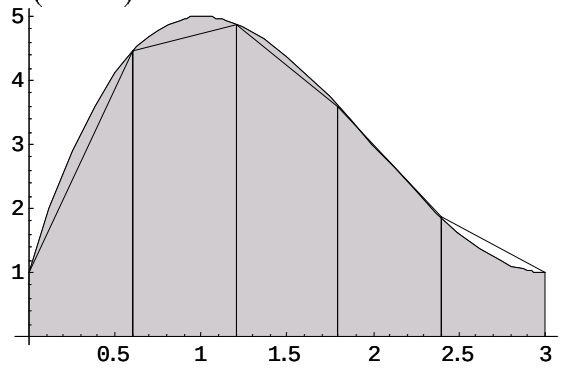
$x(x-3)^2 + 1$



0	1	1.
$\frac{1}{2}$	$\frac{33}{8}$	4.125
1	5	5.
$\frac{3}{2}$	$\frac{35}{8}$	4.375
2	3	3.
$\frac{5}{2}$	$\frac{13}{8}$	1.625
3	1	1.

$$R6 = \frac{153}{16} \approx 9.5625$$

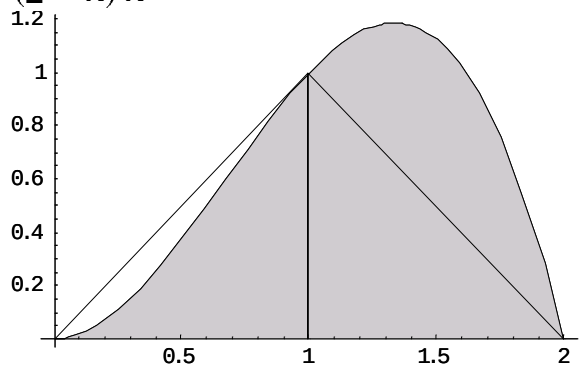
$x(x-3)^2 + 1$



0	1	1.
$\frac{3}{5}$	$\frac{557}{125}$	4.456
$\frac{6}{5}$	$\frac{611}{125}$	4.888
$\frac{9}{5}$	$\frac{449}{125}$	3.592
$\frac{12}{5}$	$\frac{233}{125}$	1.864
3	1	1.

$$T5 = \frac{237}{25} \approx 9.48$$

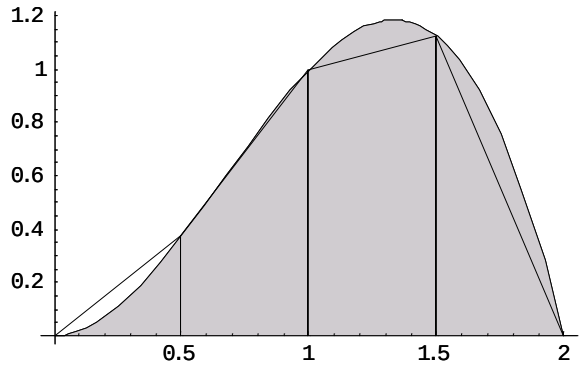
$(2-x)\,x^2$



0	0	0
1	1	1.
2	0	0.

$T2 = 1 \approx 1.$

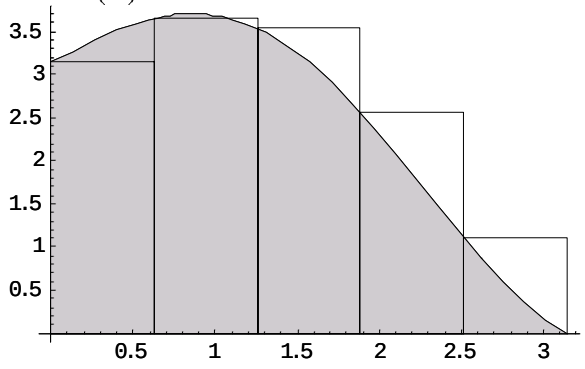
$(2-x)\,x^2$



0	0	0
$\frac{1}{2}$	$\frac{3}{8}$	0.375
1	1	1.
$\frac{3}{2}$	$\frac{9}{8}$	1.125
2	0	0.

$T4 = \frac{5}{4} \approx 1.25$

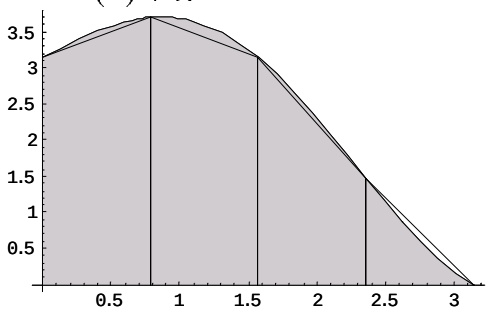
$x\cos(x)+\pi$



0	π	3.14159
$\frac{\pi}{5}$	$\pi+\frac{1}{20}\left(1+\sqrt{5}\right)\pi$	3.64991
$\frac{2\pi}{5}$	$\pi+\frac{1}{10}\left(-1+\sqrt{5}\right)\pi$	3.52991
$\frac{3\pi}{5}$	$\pi+\frac{3}{20}\left(1-\sqrt{5}\right)\pi$	2.55911
$\frac{4\pi}{5}$	$\pi+\frac{1}{5}\left(-1-\sqrt{5}\right)\pi$	1.10831
π	0	0.

$$R5 = \frac{1}{5}\pi\left(5\pi+\frac{1}{5}\left(-1-\sqrt{5}\right)\pi+\frac{3}{20}\left(1-\sqrt{5}\right)\pi+\frac{1}{10}\left(-1+\sqrt{5}\right)\pi+\frac{1}{20}\left(1+\sqrt{5}\right)\pi\right) \approx 8.78945$$

$x\cos(x)+\pi$



0	π	3.14159
$\frac{\pi}{4}$	$\pi+\frac{\pi}{4\sqrt{2}}$	3.69695
$\frac{\pi}{2}$	π	3.14159
$\frac{3\pi}{4}$	$\pi-\frac{3\pi}{4\sqrt{2}}$	1.47551
π	0	0.

$$T4 = \frac{1}{8}\pi\left(\pi+2\left(3\pi-\frac{\pi}{2\sqrt{2}}\right)\right) \approx 7.76355$$
