Limiti da risolvere con le formule degli sviluppi in serie di Taylor

Ricordando gli sviluppi in serie delle funzioni più usate calcolare i limiti in calce proposti

$$\sin(x) \to x - \frac{x^3}{6} + \frac{x^5}{120} + O(x^7)$$

$$\sin^2(x) \to x^2 - \frac{x^4}{3} + \frac{2x^6}{45} + O(x^7)$$

$$\cos(x) \to 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + O(x^7)$$

$$e^x \to 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + O(x^5)$$

$$(x+1)^{\frac{1}{n}} \to 1 + \frac{x}{n} + \frac{\left(\frac{1}{n} - 1\right)x^2}{2n} + \frac{\left(\frac{1}{n} - 2\right)\left(\frac{1}{n} - 1\right)x^3}{6n} + O(x^4)$$

$$\cos^2(x) \to 1 - x^2 + \frac{x^4}{3} - \frac{2x^6}{45} + O(x^7)$$

$$\sqrt{x+1} \to 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + O(x^5)$$

$$\log(x+1) \to x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + O(x^5)$$

$$\lim_{x\to 0} \frac{-7x^2 + e^{7x^2} - 1}{x\sin(10x) - 10x^2} =$$

$$\lim_{x\to 0} \frac{\left(\sqrt{9\sin(2x)+1} - 1\right)^2}{1 - \cos(x)} =$$

$$\lim_{x\to 0} \frac{-3x + e^{3x} - 1}{\log(\cos(2x))} =$$

$$\lim_{x\to 0} \frac{\sin^2(5x) - 25x^2}{\cos\left(2\sqrt{2}\,x^2\right) - 1} =$$

$$\lim_{x\to 0} \frac{(\sin(x) - \sin(10x))^2}{\cos(2x) - \cos(10x)} =$$

$$\lim_{x\to 0} \frac{x + \sqrt[9]{9x+1} - 1}{\sqrt{18x+1} - 1} =$$

$$\lim_{x\to 0} \frac{\sin^2(8x) - \sin(5x^2)}{\cos^2(8x) - \cos(3x)} =$$

$$\lim_{x\to 0} \frac{\sin^2(6x) - 36x^2}{64x^2 + 2\cos(8x) - 2} =$$

$$\lim_{x\to 0} \frac{-7x^2 + e^{7x^2} - 1}{x\sin(7x) - 7x^2} =$$

$$\lim_{x\to 0} \frac{\log(9x^2+1)}{\cos(9x) - \cos(x)} =$$

$$\lim_{x\to 0} \frac{-7x^2 + e^{7x^2} - 1}{x\sin(9x) - 9x^2} =$$

$$\lim_{x\to 0} \frac{\log(9x^2+1)}{\cos(9x) - \cos(2x)} =$$

$$\lim_{x\to 0} \frac{\left(\sqrt{\sin(4x)+1} - 1\right)^2}{1 - \cos(3x)} =$$

$$\lim_{x\to 0} \frac{-8x + e^{8x} - 1}{\log(\cos(5x))} =$$

$$\lim_{x\to 0} \frac{\sin^2(x) - x^2}{\cos(3x^2) - 1} =$$

$$\lim_{x\to 0} \frac{(\sin(x) - \sin(9x))^2}{\cos(8x) - \cos(9x)} =$$

$$\lim_{x\to 0} \frac{\frac{x}{5} + \sqrt[10]{2x+1} - 1}{\sqrt[6]{20x+1} - 1} =$$

$$\lim_{x\to 0} \frac{\sin^2(4x) - \sin(7x^2)}{\cos^2(4x) - \cos(2x)} =$$

$$\lim_{x\to 0} \frac{\sin^2(10x) - 100x^2}{4x^2 + 2\cos(2x) - 2} =$$

$$\lim_{x\to 0} \frac{-3x^2 + e^{3x^2} - 1}{x\sin(8x) - 8x^2} =$$

$$\lim_{x\to 0} \frac{\frac{3x}{5} + \sqrt[5]{3x+1} - 1}{\sqrt[3]{10x+1} - 1} =$$

$$\lim_{x\to 0} \frac{\sin^2(4x) - 16x^2}{16x^2 + 2\cos(4x) - 2} =$$

$$\lim_{x\to 0} \frac{-x^2 + e^{x^2} - 1}{x\sin(2x) - 2x^2} =$$

$$\lim_{x\to 0} \frac{\log(x^2+1)}{\cos(x) - \cos(3x)} =$$

$$\lim_{x\to 0} \frac{\left(\sqrt{6\sin(2x)+1} - 1\right)^2}{1 - \cos(x)} =$$

$$\lim_{x\to 0} \frac{-9x + e^{9x} - 1}{\log(\cos(6x))} =$$

$$\lim_{x\to 0} \frac{\sin^2(4x) - 16x^2}{\cos\left(\sqrt{10}\,x^2\right) - 1} =$$

$$\lim_{x\to 0} \frac{\frac{x}{6} + \sqrt[6]{x+1} - 1}{\sqrt[4]{12x+1} - 1} =$$

$$\lim_{x\to 0} \frac{\sin^2(10x) - \sin(3x^2)}{\cos^2(10x) - \cos(2x)} =$$

$$\lim_{x\to 0} \frac{\sin^2(6x) - 36x^2}{25x^2 + 2\cos(5x) - 2} =$$