Esercizi svolti con i limiti nella forma indeterminata 0/0

$$\lim_{x\to 8}\frac{\sqrt{2x+9}-5}{x-8}=$$

$$=\lim_{x\to 8}\frac{\left(\sqrt{2x+9}-5\right)\left(\sqrt{2x+9}+5\right)}{10\,(x-8)}=\lim_{x\to 8}\frac{2x-16}{10\,(x-8)}=\frac{1}{5}$$

$$\lim_{x\to 3}\frac{\sqrt{x^2-4x+9}-\sqrt{6}}{x-3}=$$

$$=\lim_{x\to 3}\frac{\left(\sqrt{x^2-4x+9}-\sqrt{6}\right)\left(\sqrt{x^2-4x+9}+\sqrt{6}\right)}{2\sqrt{6}\,(x-3)}=\lim_{x\to 3}\frac{x^2-4x+3}{2\sqrt{6}\,(x-3)}=\lim_{x\to 3}\frac{x-1}{2\sqrt{6}}=\frac{1}{\sqrt{6}}$$

$$\lim_{x\to 3}\frac{\sqrt{x^2+3x+1}-\sqrt{19}}{\sqrt{3x+8}-\sqrt{17}}=$$

$$=\lim_{x\to 3}\frac{\sqrt{x^2+3x+1}-\sqrt{19}}{\sqrt{3x+8}-\sqrt{17}}\;\frac{\sqrt{x^2+3x+1}+\sqrt{19}}{2\sqrt{19}}\;\frac{2\sqrt{17}}{\sqrt{3x+8}+\sqrt{17}}=\lim_{x\to 3}\sqrt{\frac{17}{19}}\;\frac{(x-3)\,(x+6)}{3x-9}=3\sqrt{\frac{17}{19}}$$

$$\lim_{x\to 4}\frac{\sqrt{6}\,\sqrt{x}-2\sqrt{6}}{\sqrt{x}-2}=$$

$$=\lim_{x\to 4}\frac{\sqrt{6}\,\sqrt{x}-2\sqrt{6}}{\sqrt{x}-2}\;\frac{\sqrt{6}\,\sqrt{x}+2\sqrt{6}}{4\sqrt{6}}\;\frac{4}{\sqrt{x}+2}=\lim_{x\to 4}\frac{1}{\sqrt{6}}\;\frac{6x-24}{x-4}=\sqrt{6}$$

$$\lim_{x\to 2}\frac{x^3+x^2-5x-2}{x^3+x-10}=$$

$$=\lim_{x\to 2}\frac{x^2+3x+1}{x^2+2x+5}=\frac{11}{13}$$

$$\lim_{x\to 0}\frac{3x^3+8x^2+4x}{8x-\sqrt{25-x}+5}=$$

$$=\lim_{x\to 0}\frac{x(x+2)(3x+2)}{8x-\sqrt{25-x}+5}\,\frac{10}{8x+\sqrt{25-x}+5}=$$

$$=\lim_{x\to 0}\frac{10x(x+2)(3x+2)}{64x^2+81x}=\lim_{x\to 0}\frac{10(x+2)(3x+2)}{64x+81}=\frac{40}{81}$$

$$\lim_{x\to 8}\frac{\sqrt{2x+5}-\sqrt{21}}{x-8}=$$

$$=\lim_{x\to 8}\frac{\left(\sqrt{2x+5}-\sqrt{21}\right)\left(\sqrt{2x+5}+\sqrt{21}\right)}{2\sqrt{21}\,(x-8)}=\lim_{x\to 8}\frac{2x-16}{2\sqrt{21}\,(x-8)}=\frac{1}{\sqrt{21}}$$

$$\lim_{x\to 2}\frac{\sqrt{x^2+4x+10}-\sqrt{22}}{x-2}=$$

$$=\lim_{x\to 2}\frac{\left(\sqrt{x^2+4x+10}-\sqrt{22}\right)\left(\sqrt{x^2+4x+10}+\sqrt{22}\right)}{2\sqrt{22}\,(x-2)}=\lim_{x\to 2}\frac{x^2+4x-12}{2\sqrt{22}\,(x-2)}=\lim_{x\to 2}\frac{x+6}{2\sqrt{22}}=2\sqrt{\frac{2}{11}}$$

$$\lim_{x\to 8}\frac{\sqrt{2x^2-2x+8}-2\sqrt{30}}{\sqrt{11x+10}-7\sqrt{2}}=$$

$$=\lim_{x\to 8}\frac{\sqrt{2x^2-2x+8}-2\sqrt{30}}{\sqrt{11x+10}-7\sqrt{2}}\,\frac{\sqrt{2x^2-2x+8}+2\sqrt{30}}{4\sqrt{30}}\,\frac{14\sqrt{2}}{\sqrt{11x+10}+7\sqrt{2}}=\lim_{x\to 8}\frac{7}{2\sqrt{15}}\,\frac{2(x-8)(x+7)}{11x-88}=\frac{7\sqrt{15}}{11}$$

$$\lim_{x\to 5}\frac{\sqrt{6x+7}-\sqrt{37}}{\sqrt{x}-\sqrt{5}}=$$

$$=\lim_{x\to 5}\frac{\sqrt{6x+7}-\sqrt{37}}{\sqrt{x}-\sqrt{5}}\,\frac{\sqrt{6x+7}+\sqrt{37}}{2\sqrt{37}}\,\frac{2\sqrt{5}}{\sqrt{x}+\sqrt{5}}=\lim_{x\to 5}\sqrt{\frac{5}{37}}\,\frac{6x-30}{x-5}=6\sqrt{\frac{5}{37}}$$

$$\lim_{x\to 7}\frac{x^3-6x^2-7x}{x^3-11x^2+30x-14}=$$
$$=\lim_{x\to 7}\frac{x(x+1)}{x^2-4x+2}=\frac{56}{23}$$

$$\lim_{x\to -3}\frac{5x^3+15x^2+x+3}{2x-\sqrt{13-x}+2}=$$
$$=\lim_{x\to -3}\frac{(x+3)(5x^2+1)}{2x-\sqrt{13-x}+2}-\frac{8}{2x+\sqrt{13-x}+2}=$$
$$=\lim_{x\to -3}-\frac{8(x+3)(5x^2+1)}{4x^2+9x-9}=\lim_{x\to -3}-\frac{8(5x^2+1)}{4x-3}=0$$

$$\lim_{x\to 4}\frac{\sqrt{7}\sqrt{x}-2\sqrt{7}}{x-4}=$$
$$=\lim_{x\to 4}\frac{\left(\sqrt{7}\sqrt{x}-2\sqrt{7}\right)\left(\sqrt{7}\sqrt{x}+2\sqrt{7}\right)}{4\sqrt{7}(x-4)}=\lim_{x\to 4}\frac{7x-28}{4\sqrt{7}(x-4)}=\frac{\sqrt{7}}{4}$$

$$\lim_{x\to 8}\frac{\sqrt{4x^2+4x+6}-7\sqrt{6}}{x-8}=$$
$$=\lim_{x\to 8}\frac{\left(\sqrt{4x^2+4x+6}-7\sqrt{6}\right)\left(\sqrt{4x^2+4x+6}+7\sqrt{6}\right)}{14\sqrt{6}(x-8)}=\lim_{x\to 8}\frac{4x^2+4x-288}{14\sqrt{6}(x-8)}=\lim_{x\to 8}\frac{1}{7}\sqrt{\frac{2}{3}}(x+9)=\frac{17\sqrt{\frac{2}{3}}}{7}$$

$$\lim_{x\to 3}\frac{\sqrt{2x^2-5x+1}-2}{\sqrt{10x+1}-\sqrt{31}}=$$
$$=\lim_{x\to 3}\frac{\sqrt{2x^2-5x+1}-2}{\sqrt{10x+1}-\sqrt{31}}\,\frac{1}{4}\left(\sqrt{2x^2-5x+1}+2\right)\frac{2\sqrt{31}}{\sqrt{10x+1}+\sqrt{31}}=\lim_{x\to 3}\frac{\sqrt{31}}{2}\,\frac{(x-3)(2x+1)}{10x-30}=\frac{7\sqrt{31}}{20}$$

$$\lim_{x\to 7}\frac{\sqrt{x+6}-\sqrt{13}}{\sqrt{x}-\sqrt{7}}=$$

$$=\lim_{x\to 7}\frac{\sqrt{x+6}-\sqrt{13}}{\sqrt{x}-\sqrt{7}}\;\frac{\sqrt{x+6}+\sqrt{13}}{2\sqrt{13}}\;\frac{2\sqrt{7}}{\sqrt{x}+\sqrt{7}}=\lim_{x\to 7}\sqrt{\frac{7}{13}}\,1=\sqrt{\frac{7}{13}}$$

$$\lim_{x\to -2}\frac{x^3+8x^2+7x-10}{x^3+5x^2+3x-6}=$$

$$=\lim_{x\to -2}\frac{x^2+6x-5}{x^2+3x-3}=\frac{13}{5}$$

$$\lim_{x\to 3}\frac{3x^3-10x^2+4x-3}{3x-\sqrt{172-x}+4}=$$

$$=\lim_{x\to 3}\frac{(x-3)(3x^2-x+1)}{3x-\sqrt{172-x}+4}\;\frac{26}{3x+\sqrt{172-x}+4}=$$

$$=\lim_{x\to 3}\frac{26(x-3)(3x^2-x+1)}{9x^2+25x-156}=\lim_{x\to 3}\frac{26(3x^2-x+1)}{9x+52}=\frac{650}{79}$$

$$\lim_{x\to 1}\frac{\sqrt{7x+5}-2\sqrt{3}}{x-1}=$$

$$=\lim_{x\to 1}\frac{\left(\sqrt{7x+5}-2\sqrt{3}\right)\left(\sqrt{7x+5}+2\sqrt{3}\right)}{4\sqrt{3}\,(x-1)}=\lim_{x\to 1}\frac{7x-7}{4\sqrt{3}\,(x-1)}=\frac{7}{4\sqrt{3}}$$

$$\lim_{x\to 1}\frac{\sqrt{x^2+3x+2}-\sqrt{6}}{x-1}=$$

$$=\lim_{x\to 1}\frac{\left(\sqrt{x^2+3x+2}-\sqrt{6}\right)\left(\sqrt{x^2+3x+2}+\sqrt{6}\right)}{2\sqrt{6}\,(x-1)}=\lim_{x\to 1}\frac{x^2+3x-4}{2\sqrt{6}\,(x-1)}=\lim_{x\to 1}\frac{x+4}{2\sqrt{6}}=\frac{5}{2\sqrt{6}}$$

$$\lim_{x\to 1}\frac{\sqrt{x^2+x+8}-\sqrt{10}}{\sqrt{7x+11}-3\sqrt{2}}=$$

$$=\lim_{x\to 1}\frac{\sqrt{x^2+x+8}-\sqrt{10}}{\sqrt{7x+11}-3\sqrt{2}}\,\frac{\sqrt{x^2+x+8}+\sqrt{10}}{2\sqrt{10}}\,\frac{6\sqrt{2}}{\sqrt{7x+11}+3\sqrt{2}}=\lim_{x\to 1}\frac{3}{\sqrt{5}}\,\frac{(x-1)(x+2)}{7x-7}=\frac{9}{7\sqrt{5}}$$

$$\lim_{x\to 6}\frac{\sqrt{6x+4}-2\sqrt{10}}{\sqrt{x}-\sqrt{6}}=$$

$$=\lim_{x\to 6}\frac{\sqrt{6x+4}-2\sqrt{10}}{\sqrt{x}-\sqrt{6}}\,\frac{\sqrt{6x+4}+2\sqrt{10}}{4\sqrt{10}}\,\frac{2\sqrt{6}}{\sqrt{x}+\sqrt{6}}=\lim_{x\to 6}\frac{\sqrt{\frac{3}{5}}}{2}\,\frac{6x-36}{x-6}=3\sqrt{\frac{3}{5}}$$

$$\lim_{x\to -1}\frac{x^3+7x^2+9x+3}{x^3-3x^2-2x+2}=$$

$$=\lim_{x\to -1}\frac{x^2+6x+3}{x^2-4x+2}=-\frac{2}{7}$$

$$\lim_{x\to -2}\frac{3x^3+8x^2+x-6}{-2x-\sqrt{47-x}+3}=$$

$$=\lim_{x\to -2}\frac{(x+2)(3x^2+2x-3)}{-2x-\sqrt{47-x}+3}\,\frac{14}{-2x+\sqrt{47-x}+3}=$$

$$=\lim_{x\to -2}\frac{14(x+2)(3x^2+2x-3)}{4x^2-11x-38}=\lim_{x\to -2}\frac{14(3x^2+2x-3)}{4x-19}=-\frac{70}{27}$$

$$\lim_{x\to 1}\frac{\sqrt{5x+8}-\sqrt{13}}{x-1}=$$

$$=\lim_{x\to 1}\frac{\left(\sqrt{5x+8}-\sqrt{13}\right)\left(\sqrt{5x+8}+\sqrt{13}\right)}{2\sqrt{13}\,(x-1)}=\lim_{x\to 1}\frac{5x-5}{2\sqrt{13}\,(x-1)}=\frac{5}{2\sqrt{13}}$$

$$\lim_{x\to 6}\frac{\sqrt{2x^2+3x+7}-\sqrt{97}}{x-6}=$$

$$=\lim_{x\to 6}\frac{\left(\sqrt{2x^2+3x+7}-\sqrt{97}\right)\left(\sqrt{2x^2+3x+7}+\sqrt{97}\right)}{2\sqrt{97}\,(x-6)}=\lim_{x\to 6}\frac{2x^2+3x-90}{2\sqrt{97}\,(x-6)}=\lim_{x\to 6}\frac{2x+15}{2\sqrt{97}}=\frac{27}{2\sqrt{97}}$$

$$\lim_{x\to 5}\frac{\sqrt{x^2+5x+3}-\sqrt{53}}{\sqrt{12x+4}-8}=$$

$$=\lim_{x\to 5}\frac{\sqrt{x^2+5x+3}-\sqrt{53}}{\sqrt{12x+4}-8}\;\frac{\sqrt{x^2+5x+3}+\sqrt{53}}{2\sqrt{53}}\;\frac{16}{\sqrt{12x+4}+8}=\lim_{x\to 5}\frac{8}{\sqrt{53}}\;\frac{(x-5)(x+10)}{12x-60}=\frac{10}{\sqrt{53}}$$

$$\lim_{x\to 2}\frac{\sqrt{2x+3}-\sqrt{7}}{\sqrt{x}-\sqrt{2}}=$$

$$=\lim_{x\to 2}\frac{\sqrt{2x+3}-\sqrt{7}}{\sqrt{x}-\sqrt{2}}\;\frac{\sqrt{2x+3}+\sqrt{7}}{2\sqrt{7}}\;\frac{2\sqrt{2}}{\sqrt{x}+\sqrt{2}}=\lim_{x\to 2}\sqrt{\frac{2}{7}}\;\frac{2x-4}{x-2}=2\sqrt{\frac{2}{7}}$$

$$\lim_{x\to -3}\frac{x^3+8x^2+18x+9}{x^3-2x^2-11x+12}=$$

$$=\lim_{x\to -3}\frac{x^2+5x+3}{(x-4)(x-1)}=-\frac{3}{28}$$

$$\lim_{x\to 0}\frac{2x^3+5x^2+2x}{-4x-\sqrt{9-x}+3}=$$

$$=\lim_{x\to 0}\frac{x(x+2)(2x+1)}{-4x-\sqrt{9-x}+3}\,\frac{6}{-4x+\sqrt{9-x}+3}=$$

$$=\lim_{x\to 0}\frac{6x(x+2)(2x+1)}{16x^2-23x}=\lim_{x\to 0}\frac{6(x+2)(2x+1)}{16x-23}=-\frac{12}{23}$$

$$\lim_{x\to 7}\frac{\sqrt{5x+8}-\sqrt{43}}{x-7}=$$

$$=\lim_{x\to 7}\frac{\left(\sqrt{5x+8}-\sqrt{43}\right)\left(\sqrt{5x+8}+\sqrt{43}\right)}{2\sqrt{43}\,(x-7)}=\lim_{x\to 7}\frac{5x-35}{2\sqrt{43}\,(x-7)}=\frac{5}{2\sqrt{43}}$$

$$\lim_{x\to 7}\frac{\sqrt{3x^2+7x+1}-\sqrt{197}}{x-7}=$$

$$=\lim_{x\to 7}\frac{\left(\sqrt{3x^2+7x+1}-\sqrt{197}\right)\left(\sqrt{3x^2+7x+1}+\sqrt{197}\right)}{2\sqrt{197}\,(x-7)}=\lim_{x\to 7}\frac{3x^2+7x-196}{2\sqrt{197}\,(x-7)}=\lim_{x\to 7}\frac{3x+28}{2\sqrt{197}}=\frac{49}{2\sqrt{197}}$$

$$\lim_{x\to 5}\frac{\sqrt{2x^2+7x+2}-\sqrt{87}}{\sqrt{11x+9}-8}=$$

$$=\lim_{x\to 5}\frac{\sqrt{2x^2+7x+2}-\sqrt{87}}{\sqrt{11x+9}-8}\,\frac{\sqrt{2x^2+7x+2}+\sqrt{87}}{2\sqrt{87}}\,\frac{16}{\sqrt{11x+9}+8}=\lim_{x\to 5}\frac{8}{\sqrt{87}}\,\frac{(x-5)(2x+17)}{11x-55}=\frac{72\sqrt{\frac{3}{29}}}{11}$$

$$\lim_{x\to 6}\frac{4x^3-20x^2-26x+12}{3x-\sqrt{406-x}+2}=$$

$$=\lim_{x\to 6}\frac{2(x-6)(2x^2+2x-1)}{3x-\sqrt{406-x}+2}\,\frac{40}{3x+\sqrt{406-x}+2}=$$

$$=\lim_{x\to 6}\frac{80(x-6)(2x^2+2x-1)}{9x^2+13x-402}=\lim_{x\to 6}\frac{80(2x^2+2x-1)}{9x+67}=\frac{6640}{121}$$